

دورة سنة 2013 العادية	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ساعتان	الجمعة 28 حزيران 2013

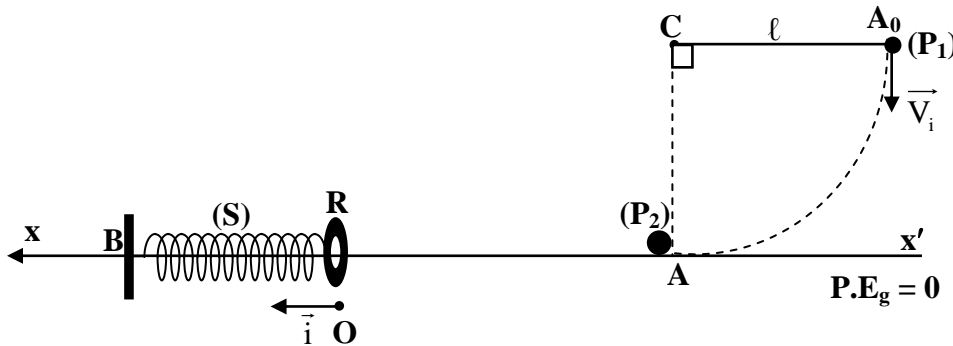
This exam is formed of three obligatory exercises in 3 pages numbered from 1 to 3
The use of non-programmable calculator is recommended

First exercise: (7 points)

Collisions and mechanical oscillator

A – Collision

A pendulum is formed of a massless and inextensible string of length $\ell = 1.8$ m, having one of its ends C fixed to a support while the other end carries a particle (P_1) of mass $m_1 = 200$ g .
The pendulum is stretched horizontally. The particle (P_1) at A_0 is then launched vertically downward with a velocity \vec{V}_i of magnitude $V_i = 8$ m/s.
At the lowest position A, (P_1) enters in a head-on perfectly elastic collision with another particle (P_2) of mass $m_2 = 300$ g initially at rest. Neglect all frictional forces.



Take:

- the horizontal plane passing through A as a gravitational potential energy reference;
 - $g = 10$ m/s².
- 1) a) Calculate the mechanical energy of the system [pendulum, Earth] at the instant of launching (P_1) at A_0 .
b) Determine the magnitude V_1 of the velocity \vec{V}_1 of (P_1) just before colliding with (P_2).
 - 2) a) Name the physical quantities that are conserved during this collision.
b) Show that the magnitude V_2' of the velocity \vec{V}_2' of (P_2), just after collision, is 8 m/s.

B – Mechanical oscillator

A horizontal spring (S), of negligible mass and of stiffness $K = 120$ N/m, is connected at one of its ends B to a fixed support while the other end is attached to a ring R.
(P_2) moves on the horizontal path AB until it hits the ring R at point O; (P_2) sticks to R forming a solid (P), considered as a particle, of mass $m = 1.2$ kg. Thus (P) and the spring (S) form a horizontal mechanical oscillator of center of inertia G; G moves without friction on a horizontal axis $x'Ox$ along AB. Just after collision and at the initial instant $t_0 = 0$, G coincides with O, the equilibrium position of (P), and has a velocity $\vec{V}_0 = V_0 \vec{i}$ with $V_0 = 2$ m/s.

At an instant t , the abscissa of G is x and the algebraic value of its velocity is $v = \frac{dx}{dt}$.

- 1) Write down the expression of the mechanical energy of the system (oscillator, Earth) at an instant t , in terms of K , m , x and v .
- 2) Derive the differential equation in x that describes the motion of G and deduce the nature of its motion.
- 3) Knowing that the solution of this differential equation is $x = X_m \cos(\sqrt{\frac{K}{m}} t + \varphi)$, determine the values of the constants X_m and φ .

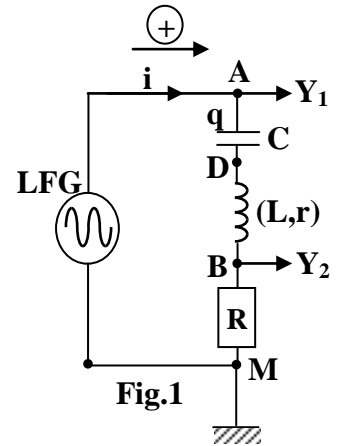
Second exercise: (7 points)

Determination of the characteristics of a coil and a capacitor

The aim of this exercise is to determine the characteristics of a capacitor and a coil.

In order to determine these characteristics, we connect in series a capacitor of capacitance C , a coil of inductance L and of resistance r , a resistor of resistance $R = 20 \Omega$ and a low frequency generator (LFG) delivering an alternating sinusoidal voltage u of constant maximum value U_m and of adjustable frequency f .

The circuit thus formed, carries an alternating sinusoidal current i (Fig. 1).



An oscilloscope is connected to display the voltage $u = u_{AM}$ across the terminals of the (LFG) on channel (Y_1) and the voltage u_{BM} across the terminals of the resistor (R) on channel (Y_2).

The settings of the oscilloscope are:

- horizontal sensitivity: $S_h = 2 \text{ ms/div}$;
- vertical sensitivity: - On (Y_1) : $S_{V1} = 2 \text{ V/div}$;
- On (Y_2) : $S_{V2} = 0.25 \text{ V/div}$.

A – For a given value f_0 of the frequency f we observe on the screen of the oscilloscope the waveforms represented by figure 2.

- 1) Determine f_0 and the proper angular frequency ω_0 .
- 2) Determine the maximum value U_m of u and the maximum current I_m of i .
- 3) a) The waveforms show that a physical phenomenon takes place in the circuit. Name this phenomenon. Justify.
b) Deduce the relation between L and C .
- 4) The circuit between A and M is equivalent to a resistor of resistance $R_t = R + r$. Determine R_t and deduce r .

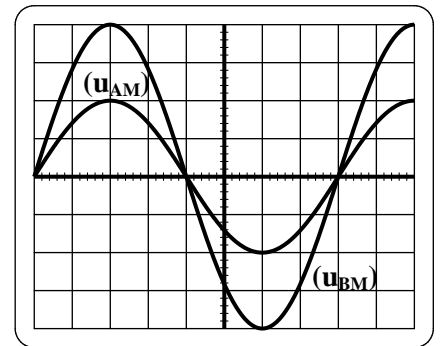


Fig.2

B – The coil in the circuit of figure 1 is replaced by a resistor r_1 of resistance $r_1 = 60 \Omega$ (figure 3).

The voltage across the terminals of the generator is $u = u_{AM} = U_m \cos \omega_0 t$. On the screen of the oscilloscope, we observe the waveforms represented by figure 4. The settings of the oscilloscope are not changed.

- 1) Using the waveforms of figure 4:
 - a) tell why the voltage u_{AM} lags behind u_{BM} ;
 - b) calculate the phase difference φ between u_{AM} and u_{BM} ;
 - c) determine the expressions of u_{BM} and of u_{AM} as a function of time t .
- 2) Write down the expression of i as a function of time t .
- 3) The voltage across the terminal of the capacitor is:

$$u_C = u_{AD} = \frac{8.9 \times 10^{-5}}{C} \sin \left(125\pi t + \frac{\pi}{4} \right); \text{ [u in V and t in s].}$$

By applying the law of addition of voltages and giving t a particular value, determine the value of C .

C – Use the relation found in part [A-3 (b)] , calculate L .

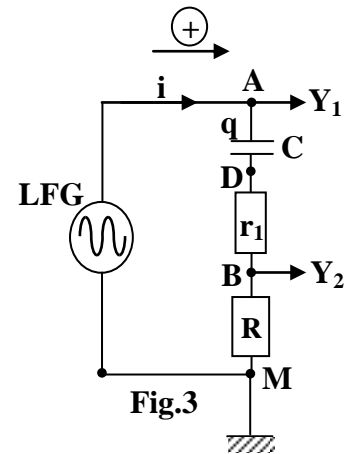


Fig.3

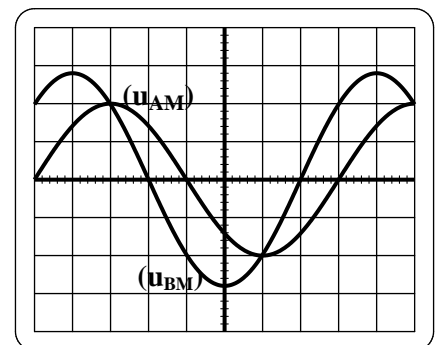


Fig.4

Third exercise: (6 points)

Dating by Carbon 14

The radioactive carbon isotope $^{14}_6\text{C}$ is a β^- emitter. In the atmosphere, $^{14}_6\text{C}$ exists with the carbon 12 in a constant ratio.

When an organism is alive it absorbs carbon dioxide that comes indifferently from carbon 12 and carbon 14. Just after the death of an organism, this absorption stops and carbon 14, that it has, disintegrate with a half life $T = 5700$ years.

In living organisms, the ratio of the number of carbon 14 atoms to that of the number of carbon 12 atoms

$$\text{is: } r_0 = \frac{\text{initial number of carbon 14 atoms}}{\text{number of carbon 12 atoms}} = \frac{N_0(^{14}\text{C})}{N'(^{12}\text{C})} = 10^{-12} .$$

After the death of an organism by a time t , the ratio of the number of carbon 14 atoms to that of the

$$\text{number of carbon 12 atoms becomes: } r = \frac{\text{remaining number of carbon 14 atoms}}{\text{number of carbon 12 atoms}} = \frac{N(^{14}\text{C})}{N'(^{12}\text{C})} .$$

1) The disintegration of $^{14}_6\text{C}$ is given by: $^{14}_6\text{C} \rightarrow \frac{A}{Z}\text{N} + \beta^- + \text{}^0_0\bar{\nu}$.

Calculate Z and A , specifying the laws used.

2) Calculate, in year^{-1} , the radioactive constant λ of carbon 14.

3) Using, the law of radioactive decay of carbon 14, $N(^{14}\text{C}) = N_0(^{14}\text{C}) \times e^{-\lambda t}$.

Show that $r = r_0 e^{-\lambda t}$.

4) Measurements of $\frac{r}{r_0}$, for specimens a, b and c, are given in the following table:

ratio	specimen a	specimen b	specimen c
$\frac{r}{r_0}$	0.914	0.843	0.984

a) Specimen b is the oldest. Why?

b) Determine the age of specimen b.

5) a) Calculate the ratio $\frac{r}{r_0}$ for $t_0 = 0$, $t_1 = 2T$, $t_2 = 4T$ and $t_3 = 6T$.

b) Trace then the curve $\frac{r}{r_0} = f(t)$ by taking the following scales:

- On the abscissa axis: $1 \text{ cm} \rightarrow 2T$
- On the ordinate axis: $1 \text{ cm} \rightarrow \frac{r}{r_0} = 0.2$

c) To determine the date of death of a living organism, it is just enough to measure $\frac{r}{r_0}$.

Explain why we cannot use the traced curve to determine the date of the death of an organism that died several millions years ago.

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Solutions

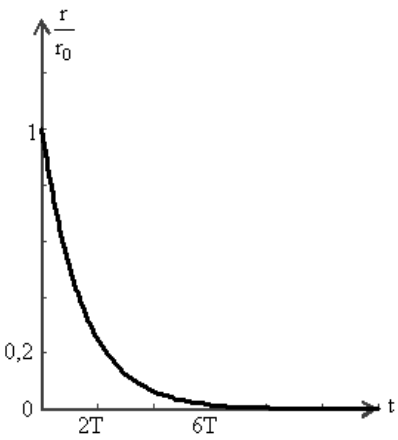
First exercise (7 points)

Part of the Q	Answer	Mark
A-1-a	$ME_i = KE_i + PE_{g_i} = \frac{1}{2}m_1 V_i^2 + m_1 g l = 0.5 \times 0.2 \times 64 + 0.2 \times 10 \times 1.8 = 10 \text{ J}$	0.75
A-1-b	Since there is no friction then ME is conserved so $ME_i = 10 = ME_A = \frac{1}{2} m_1 V_1^2 + 0$ $\Rightarrow 10 = 0.1 V_1^2 + 0 \Rightarrow V_1 = 10 \text{ m/s.}$	0.75
A-2.a	The linear momentum and the kinetic energy.	0.50
A-2.b	Conservation of linear momentum: $m_1 \vec{V}_1 + 0 = m_1 \vec{V}_1' + m_2 \vec{V}_2'$ but no deviation (head-on) $\Rightarrow m_1 V_1 + 0 = m_1 V_1' + m_2 V_2' \Rightarrow m_1 (V_1 - V_1') = m_2 V_2' \dots (1)$ collision is elastic: $\frac{1}{2} m_1 V_1^2 = \frac{1}{2} m_1 (V_1')^2 + \frac{1}{2} m_2 (V_2')^2$ $\Rightarrow m_1 [V_1^2 - (V_1')^2] = m_2 (V_2')^2 \dots (2)$ Divide (2) by (1) we get: $V_1 + V_1' = V_2' \dots (3)$ Equations (1) and (3) give: $V_2' = 8 \text{ m/s.}$	1.5
B-1	$M.E = \frac{1}{2} kx^2 + \frac{1}{2} mV^2.$	0.5
B-2	$M.E = \text{constant}$ $\Rightarrow \frac{dM.E}{dt} = 0$ $\Rightarrow kxx' + mVV' = 0 ; V = x' \neq 0 \text{ and } V' = x''$ $\Rightarrow x'' + \left(\frac{K}{m}\right)x = 0.$ This differential equation has the form of $x'' + \omega_0^2 x = 0 ;$ The motion is simple harmonic.	1
B-3	$ME_{x=0} = ME_{x=x_m} \Rightarrow \frac{1}{2} mV_o^2 + \frac{1}{2} Kx_o^2 = \frac{1}{2} KX_m^2$ $\frac{1}{2} \times 1.2 \times 2^2 + 0 = \frac{1}{2} \times 120 \times X_m^2 \Rightarrow X_m = 0.2 \text{ m} = 20 \text{ cm.}$ $x = X_m \cos\left(\sqrt{\frac{K}{m}} t + \varphi\right)$ at $t = 0 \text{ s, } x = 0 \Rightarrow 0 = X_m \cos \varphi \Rightarrow \cos \varphi = 0 \Rightarrow \varphi = \pm \frac{\pi}{2}$ but at $t = 0$ we have $v = V_o = -X_m \sin \varphi > 0 \Rightarrow \varphi = -\frac{\pi}{2} \text{ rd}$	1 1

Second exercise (7 points)

Part of the Q	Answer	Mark
A-1	$T_0 = 8 \times 2 = 16 \text{ ms} \Rightarrow f_0 = \frac{1}{T_0} = 62.5 \text{ Hz}$ and $\omega_0 = 2\pi f_0 = 125\pi \text{ rd/s}$.	0.5 ; 0.25 0.25
A-2	$U_m = 2 \times 2 = 4\text{V}$ $U_{Rm} = 4 \times 0.25 = 1\text{V} \Rightarrow I_m = \frac{U_{Rm}}{R} = \frac{1}{20} = 0.05 \text{ A}$	0.25 0.75
A-3-a	Current resonance, since u_{AM} and $u_{BM} = Ri$ are in phase	0.25 ; 0.25
A-3-b	Since we have current resonance then $LC\omega_0^2 = 1$ so $LC = 6.49 \times 10^{-6}$.	0.25 ; 0.5
A-4	$U_m = I_m \times R_t \Rightarrow R_t = \frac{4}{0.05} = 80\Omega \Rightarrow r = 80 - 20 = 60 \Omega$	0.25; 0.25
5- B-1-a	Since u_{BM} reaches its maximum before that of u_{AM} .	0.25
B-1-b	$2\pi \text{ rd} \rightarrow 8 \text{ div} \rightarrow T_0$ $\varphi \rightarrow 1 \text{ div} \Rightarrow \varphi = \frac{2\pi}{8} = \frac{\pi}{4} \text{ rd}$.	0.5
B-1-c	$U_{BM\max} = 2.8 \times 0.25 = 0.7 \text{ V}$ $\Rightarrow u_{BM} = 0.7 \cos\left(125\pi t + \frac{\pi}{4}\right)$ (u_{BM} in V, t in s) $U_m = 2 \times 2 = 4\text{V} \Rightarrow u = 4 \cos 125\pi t$ (u in V, t in s).	0.50 0.25
B-2	$I_m = \frac{U_{BM\max}}{R} = \frac{2.8 \times 0.25}{20} = 0.035 \text{ A}$ $\Rightarrow i = 0.035 \cos\left(125\pi t + \frac{\pi}{4}\right)$ (i in A, t in s).	0.5
B-3	The law of addition of voltages gives : $u_{AM} = u_{AD} + u_{DB} + u_{BM}$ $4 \cos 125\pi t = \frac{8.9 \times 10^{-5}}{C} \sin\left(125\pi t + \frac{\pi}{4}\right) + 80 \times 0.035 \cos\left(125\pi t + \frac{\pi}{4}\right)$ For $125\pi t = \frac{\pi}{2}$; $0 = \frac{8.9 \times 10^{-5}}{C} \cos \frac{\pi}{4} - 2.8 \sin \frac{\pi}{4} \Rightarrow \frac{8.9 \times 10^{-5}}{C} = 2.8$ $C = \frac{8.9 \times 10^{-5}}{2.8} = 32 \times 10^{-6} \text{ F} = 32 \mu\text{F}$.	1
C	$LC = 6.49 \times 10^{-6} \Rightarrow L \times 32 \times 10^{-6} = 6.49 \times 10^{-6} \Rightarrow L = \frac{6.49}{32} = 0.2 \text{ H}$	0.25

Third exercise (6 points)

Part of the Q	Answer	Mark
1	${}^{14}_6\text{C} \rightarrow {}^0_{-1}\text{e} + {}^A_Z\text{X} + {}^0_0\bar{\nu}$ law of conservation of mass number: $14 = 0 + A + 0$ then $A = 14$ law of conservation of charge number: $6 = 0 - 1 + Z + 0$ then $Z = 7$.	0.25 ; 0.25 ; 0.25 ; 0.25.
2	$\lambda = \frac{0.693}{T} = 1.216 \times 10^{-4} \text{ year}^{-1}$	0.75
3	$r = \frac{N({}^{14}\text{C})}{N'({}^{12}\text{C})} = \frac{N_0({}^{14}\text{C}) \times e^{-\lambda t}}{N'({}^{12}\text{C})}$ with $r_0 = \frac{N_0({}^{14}\text{C})}{N'({}^{12}\text{C})}$, we can write $r = r_0 e^{-\lambda t}$.	0.75
4-a	$\frac{r}{r_0} = e^{-\lambda t}$ as t increases then $e^{-\lambda t}$ decreases then $\frac{r}{r_0}$ decreases Since specimen b has the lowest ratio then it is the oldest.	0.5
4-b	$\frac{r}{r_0} = e^{-\lambda t} = 0.843$ then $\ln 0.843 = -\lambda \times t$ thus the age of the specimen is $t = \frac{-0.171}{-1.216 \times 10^{-4}} = 1406.25 \text{ years}$.	1
5-a	the ratio $\frac{r}{r_0} = e^{-\lambda t}$ for $t_0 = 0$ $\frac{r}{r_0} = 1$; for $t = 2T$ then $\frac{r}{r_0} = 0.25$; for $t = 4T$ then $\frac{r}{r_0} = 0.0625$ for $t = 6T$ then $\frac{r}{r_0} = 0.015625$.	1
5-b		0.5
5-c	Since after millions of years the ratio $\frac{r}{r_0}$ becomes zero so we cannot determine the age of such organism.	0.5